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The siphon

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The siphon has been known since early times as a simple and effective device for transferring liquid from a higher to a lower level and is still widely used. Despite this, considerable confusion exists as to how the siphon works and a survey of elementary textbooks (here 'elementary' refers to GCE O and A level standard) yields a number of conflicting accounts. Many are in terms of hydrostatic principles which cannot apply and, in addition, atmospheric pressure is postulated as an essential agency in most accounts, including those given by the standard dictionaries. Few books point out that it is the cohesion of the liquid rather than the pressure of an external atmosphere which is crucial to the working of a siphon. Some American college texts discuss the siphon and give a correct theory although generally in terms of non-viscous fluids. It is unusual for more advanced texts to discuss the siphon, for it is only a particular case of the general problem of pipe flow which has been the subject of thorough investigation. We have written this article because we are concerned that misleading interpretations of the siphon are still current even though much is known about the physics of pipe flow and the properties of liquids.

We begin with a survey of the 'hydrostatic' theories, following this with the dynamic theory for an 'ideal' nonviscous liquid. We modify our 'ideal' theory to take account of the viscous nature of liquids and then give some experimental results. Finally we attempt to analyse the factors underlying the working of the siphon and suggest a possible theory.

Hydrostatic theories

A typical account of the siphon given in elementary text books might read as follows.

Referring to figure 1, p_B equals p_0, where p_0 is the pressure of the surrounding atmosphere and p_B is the pressure at point B in the liquid. Since B and E lie on the same horizontal level (strictly, on the same gravitational equipotential), p_B equals p_E. The pressure p_F of the liquid at F equals $p_E + \rho gh$ and therefore also equals $p_0 + \rho gh$ where ρ is the density of the liquid. Thus p_F is greater than the external pressure, and the liquid flows out of the tube. In order to prevent a vacuum forming in the tube more liquid, pushed in by the atmospheric pressure, enters at A. Another common account begins by saying that p_B equals p_0 and p_F equal p_0. Then, by consideration of heights of liquid, it is argued that the pressure at C must be greater than the pressure at D, and this pressure difference causes the liquid to flow. This account is incorrect. If the liquid is stationary p_C must always equal p_D. If the liquid flows and we apply the principles of fluid dynamics, we still find that the pressures at points on the same horizontal level are equal if the pipe is of uniform bore and we neglect viscous losses.

The first account given above is correct when no flow occurs but its limitations as a theory for a working siphon are obvious since it describes a static condition. The siphon is essentially a device involving fluid flow and it is desirable that our theory should allow us to make some quantitative predictions such as, for example, the dependence of the rate of flow of liquid on the height h.

Dynamic theory

The discussion below considers the flow of fluids in

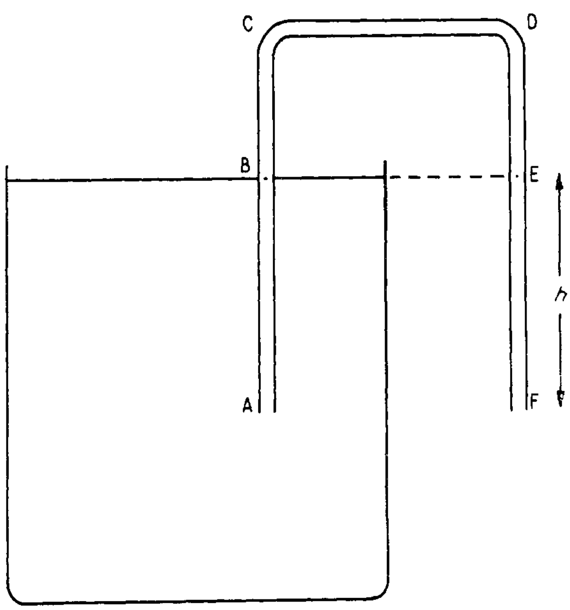

Figure 1 *Diagram of a siphon.*

tubes. Provided there are no leaks the mass of fluid crossing each section of a tube per unit time must be the same. Conservation of mass is expressed by the equation of continuity

$$\rho A v = \text{constant} \tag{1}$$

where A is the area of a cross section of the tube and ρ and v are the values of the density and the velocity of the fluid flowing normal to the cross section. For an incompressible fluid ρ is constant and the equation of continuity reduces to

$$Av = \text{constant}. \tag{2}$$

(i) For an ideal fluid

Because all real fluids exhibit viscosity there is an inevitable dissipation of energy in fluid flow but let us for a moment consider an 'ideal' non-viscous and incompressible fluid. We assume that its flow through a tube is steady and irrotational (which excludes the possibility of a vortical motion) and that there is no possibility of heat exchange between the fluid and its surroundings. Then the energy of the fluid flowing along the tube will be conserved. This is expressed mathematically by the Bernoulli equation

$$p + \tfrac{1}{2}\rho v^2 + \rho g h = \text{constant}. \tag{3}$$

The parameters are again average values for the cross section, p representing the pressure in the fluid and h the elevation of the tube relative to some fixed level.

Suppose that liquid is siphoned from a vessel with a cross sectional area which is large compared with that of the siphon tube. Then the downward velocity of the surface of the liquid in the vessel is effectively zero (by equation (2)). If the siphon tube is of uniform bore, (again by equation (2)), the velocity of flow v will be the same throughout the tube. We can apply equation (3) anywhere inside the vessel–siphon system. Thus

$$p_0 = p_B + \tfrac{1}{2}\rho v^2$$

where the left hand side of the equation has been evaluated at the surface of the liquid in the vessel and the right hand side has been evaluated at B.

Energy conservation between B and F implies that

$$p_B + \tfrac{1}{2}\rho v^2 = p_F + \tfrac{1}{2}\rho v^2 - \rho g h.$$

At F the tube is open to the external pressure so that

$$p_F = p_0.$$

Combining these three equations we have

$$v^2 = 2gh \tag{4}$$

that is the fluid emerges from the siphon with just the velocity it would acquire by falling through a height h.

It is of interest to note that the pressures at points inside the tube are all less than the external pressure, with the exception of the pressure at F, which is equal to the external pressure. The pressures in the dynamic situation are less than those in the static situation by an amount $\frac{1}{2}\rho v^2$; in particular, for the static case, there is a pressure head of $\rho g h$ acting while for the dynamic case the effective pressure head is $\rho g h - \frac{1}{2}\rho v^2$.

(ii) For a real liquid

When we consider the flow of a real fluid we immediately find complications because there are two major types of flow possible: a smooth laminar flow and a turbulent one. Osborne Reynolds distinguished between these two types of flow in his experiments on the flow of water through pipes and he established a criterion, the non-dimensional Reynolds Number, in terms of which the flow might be described.

Below a certain critical Reynolds Number of about 2000, the flow in the pipe remained laminar along its whole length. Above that number there was a transition to turbulent flow at some point along the pipe. The Reynolds Number is given by

$$R = \frac{v d \rho}{\eta} \tag{5}$$

where v and d are respectively a velocity and a length representative of the flow, η is the coefficient of viscosity and ρ is the density of the fluid. For the case of pipe flow v is taken as the volume rate of flow divided by the cross sectional area of the pipe, and d is the diameter of the pipe.

The working siphon is an example of flow in a U shaped pipe and for a given liquid and pipe there is the possibility of either laminar or turbulent flow depending on the velocity. In both cases energy will be lost by the liquid because of its viscosity and the Bernoulli equation, which is a statement of the conservation of energy of a non-viscous liquid, cannot be applied. If we wish to calculate the average velocity of a viscous liquid in a siphon and yet maintain the form of the Bernoulli equation, we may do so by introducing a loss of head h_L which corresponds to the energy losses suffered by the liquid. The average velocity instead of being given by

$$v^2 = 2gh$$

is now given by

$$v^2 = 2g(h - h_L). \tag{6}$$

The loss of head in *laminar flow* is due solely to skin friction whereas in *turbulent flow* there are, in addition, losses associated with the pipe 'fittings'. In this case the 'fittings' are the entrance and bends.

For both types of flow, the loss of head due to skin friction is given by Darcy's equation

$$h_f = f \frac{l}{d} \frac{v^2}{2g} \tag{7}$$

where f is a dimensionless friction factor and l the length of pipe. The friction factor is a function of the

Reynolds Number and, in the case of turbulent flow, also depends on the roughness of the inside of the pipe.

(*a*) *Laminar flow*. The average velocity for laminar flow in a pipe of circular cross section is given by the Poiseuille equation, here expressed in the form

$$v = \frac{d^2}{32\eta} \times \text{(pressure gradient along the pipe)}$$

$$= \frac{d^2}{32\eta} \frac{\rho g h - \frac{1}{2}\rho v^2}{l}.$$

After multiplying through by v and some rearrangement we may write

$$v^2 = \frac{2gh}{1 + (64/R)\, l/d} \tag{8}$$

Because the losses in laminar flow are solely due to skin friction, $h_L = h_f$. Hence, using equations (6) and (7),

$$v^2 = 2gh - \frac{flv^2}{d}$$

giving

$$v^2 = \frac{2gh}{1 + fl/d}. \tag{9}$$

By comparison of equations (8) and (9) we see that

$$f = \frac{64}{R}. \tag{10}$$

(*b*) *Turbulent flow*. We shall consider only smooth pipes in which the friction factor is sufficiently accurately given by Blasius' empirical equation (valid for $3000 < R < 10^5$)

$$f = 0{\cdot}3164\, R^{-\frac{1}{4}}. \tag{11}$$

The additional loss in head due to the pipe fittings, which occurs only for turbulent flow, has been found to vary approximately as $v^2/2g$. The net loss of head caused by fittings is given by

$$h_F = (k_1 + k_2 + k_n + \ldots)\frac{v^2}{2g} \tag{12}$$

where k_n is the loss coefficient of an individual fitting. Empirical values of k_n can be found in tables.

For turbulent flow, therefore, the total loss of head is given by

$$h_L = h_f + h_F. \tag{13}$$

(*c*) *Inlet length*. The formulae for head losses due to friction given in the two previous sections are valid only for fully developed laminar or turbulent flow. Close to the entrance the presence of the pipe walls has a negligible effect on the central core of the fluid but, further down the pipe the walls affect the viscosity more and more of the fluid until eventually the final fully developed flow is achieved.

The inlet length is defined as the length beyond which the flow has become fully developed. For laminar flow it is, according to Langhaar, 0·057 *Rd* (see Massey 1968). For turbulent flow the inlet length depends on the conditions at the entrance and theory suggests it is a slowly varying function of Reynolds number. However, for this latter case no consistent experimental results have been found for the value of the inlet length, but it would appear reasonable to take it as 50*d*.

A value for f, taking into account the entrance length in laminar flow, has been calculated by Atkinson and Goldstein (see Goldstein Vol 1, 1938) and is given by

$$f = \frac{64}{R} + 1{\cdot}41\frac{d}{l}. \tag{14}$$

A description of a different approach to the problem of corrections to the Poiseuille equation is given by Newman and Searle (1957).

Experimental results

In our experiments we used siphon tubes of diameters ranging from approximately 0·15 cm up to 2·0 cm and we were able to obtain both laminar and turbulent flow. The basic technique was to siphon water out of a vessel of large cross sectional area compared with that of the siphon tube. The rate of flow of water through the siphon for a given head h was found by collecting water in a calibrated vessel for a known period of time. The head h was varied by raising or lowering the siphon tube.

Narrow siphon tubes, of diameter 0·173 cm and 0·312 cm, were made from glass tubing bent through two right angles to form U tubes with equal arms of lengths of approximately 50 cm. Water was siphoned from a plastic bin of diameter 40 cm and was collected in standard measuring cylinders.

The wider siphons, of approximate diameters 1·4 cm and 2·0 cm, were made from straight lengths of copper tubing joined by standard 'Yorkshire' right-angle fittings. The siphons had equal arms of lengths of approximately 90 cm. A large roof storage tank (approximate areal dimensions 2 m by 1½ m) acted as a reservoir and we measured the amount of water collected in a calibrated plastic bin.

All distances were measured with a metre rule except for the tube diameters which were measured with a travelling microscope.

The results for the friction factor for flow through the two glass tubes are shown in figure 2 together with the relevant theoretical curves. It may be seen that our experimental values are consistently higher than theory would indicate. For $R < 2000$ inconsistencies in the tube bore, for example narrowing at the bends, is a possible explanation for this divergence and for $R > 2000$ it is likely that our flow was no longer laminar.

In the copper tubes Reynolds numbers ranging from

1×10^4 to 3×10^4 were obtained. The expected losses arising from turbulent flow through the siphon were obtained as follows.

By substituting equations (7) and (12) into (13) we obtain

$$h_L = f\frac{l}{d} + \sum k_n \frac{v^2}{2g}.$$

Therefore, by equation (6)

$$\frac{v^2}{2g} = h - \left(f\frac{l}{d} + \sum k_n\right)\frac{v^2}{2g}$$

giving

$$\frac{2gh}{v^2} = 1 + f\frac{l}{d} + \sum k_n.$$

In our case $\Sigma k_n = 2{\cdot}2$ (k for each bend being taken as 0·6, k for the entry as 1·0). f was calculated using equation (11).

The experimental values of $2gh/v^2$ were lower than expected, by 20% for the narrower tube and by 10% for the wider tube. Since the entry lengths occupy a considerable proportion of the total length of the tube and the point in the tube at which the flow becomes turbulent is uncertain, these results are perhaps not surprising.

Conditions for a working siphon

That a liquid cannot sustain a tension and that the working of a siphon depends on an external pressure appear to be two popular misconceptions.

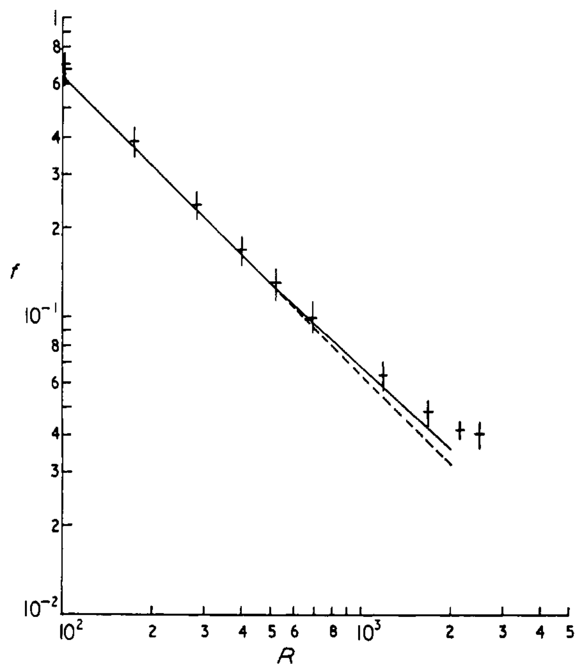

Figure 2 Comparison of measured values of the friction factor f and two theoretical estimates: broken curve, Poiseuille's law $f = 64/R$; and full curve, Atkinson and Goldstein equation (equation (14)) $f = 64/R + 1{\cdot}41\,d/l$.

Trevena (1967) has written "It is not generally appreciated that liquids can, under appropriate conditions, sustain very considerable tensions". This statement should not surprise us unduly, for we accept that the cohesive forces between molecules account for the existence of, for example, the liquid phase and the surface properties of liquids. Since the liquid in a siphon can withstand tension, the barometric height should not place a limit on the 'uptake height' (ie the height BC) for which a siphon will work. Nokes (1948), for example, has reported the successful operation of a mercury siphon with an uptake height of 80 to 84 cm in air at atmospheric pressure and Ashhurst (1966) states that such a siphon has been made to work with an uptake of 150 cm.

Several questions immediately come to mind. Is there a limit on the uptake height? Why is it a useful rule of thumb not to attempt to operate a siphon with an uptake height of the order of or exceeding the barometric height? Does the pressure of the external atmosphere play any part in the working of the siphon?

The amount of tension that a liquid flowing in a pipe can sustain, without forming cavities that will grow and disrupt the flow depends on a number of factors. Temperley and Chambers (1946) for example found that tap water flowing through a constriction cavitated at a critical tension of 0·05 atm. However, the critical tension increased to 0·22 atm if the tap water had already been passed through the apparatus once and further increased to 0·27 atm if the water had passed through the apparatus three times. Batchelor (1967) gives a possible explanation of how cavitation occurs in situations similar to those of the above experiment. He assumes that there are tiny air bubbles present in the water which grow continuously when the pressure falls sufficiently below the saturated vapour pressure of water. If water has been boiled or subjected to a large positive pressure, which presumably removes all the air bubbles although leaving the water saturated with air, it can withstand considerable tensions, certainly above 20 atm. Temperley has calculated that water should have a critical tension of 500 atm and this value is reduced by less than 0·5% if the water is assumed saturated with (dissolved) air at a pressure of 1 atm (see Trevena 1967). A wide range of experimental values for the critical tension of water have been reported (see Trevena 1967) but Briggs (1950) has obtained a value as high as 277 atm for boiled water in a capillary tube open to the atmosphere. It is interesting to note that Trevena (1967) points out that these experimental values for the critical tension are a measure of the 'weakest link' in the liquid container system, so that they depend on the material and nature of the walls of the container. It is clear then that liquids can support very considerable tensile stresses. In practice, of course, we may expect that the liquids being siphoned will contain dissolved air, foreign matter and tiny air bubbles and in these circumstances it is a useful rule to assume that the critical pressure is equal to the vapour pressure of the liquid.

The effect of an external pressure is to compress the liquid column. For a viscous liquid moving through the siphon, the pressure at D on the diagram is

$$p_D = p_0 - \tfrac{1}{2}\rho v^2 - \rho g (h_D - h_{LD})$$

where v is the observed velocity of flow, h_{LD} represents the loss in head effective at D caused by friction losses, and h_D is the loss in head due to elevation. The larger p_0 is, the greater h_D can become before p_D becomes less than the saturated vapour pressure and the column becomes liable to breakage. If there are no effective nuclei present in the liquid–siphon system, we would not expect the external pressure to play any appreciable part in the working of the siphon. Again Nokes (1948) reported operating siphons with water, dibutyl phthalate and mercury under 'vacuum' conditions; in fact the siphons functioned under the vapour pressures of the liquids in them which, at 20°C, would range from 1·75 cm Hg for water down to $1{\cdot}2 \times 10^{-4}$ cm Hg for mercury. Nokes ensured that his apparatus was clean before use, and after filling the apparatus boiled the liquid 'to remove dissolved gas'. He states that the chief disturbing factors which tend to break the liquid column of a siphon working under 'vacuum' conditions are gas dissolved in the liquid, adherent gas on the walls of the tube, mechanical shock and turbulent flow of the liquid.

Explanation of the siphon

By now it should be clear that, despite a wealth of tradition, the basic mechanism of a siphon does not depend upon atmospheric pressure.

Nokes (1948) cites the following explanation, which is also given in the books by Abbott (1963) and Ashhurst (1966). In a siphon working under vacuum conditions liquid will flow through the siphon if the gravitational force acting on the liquid in the downtake tube (ie the tube from D to F) is greater than the gravitational force acting on the liquid in the uptake tube. The liquid column, dragged through the tube by its own weight, is in a state of tension and it is because of the cohesive forces between the molecules of the liquid that the column remains unbroken. The effect of an external pressure is to compress the liquid column rendering it less liable to breakage. It is a helpful analogy to imagine the liquid in the vessel and the siphon being replaced by a chain.

An explanation in terms of fluid dynamics would run thus. Imagine a vessel with liquid in it and a siphon tube, filled with liquid, with one end immersed in the liquid in the vessel and the other end at F blocked (see figure 1). When the obstruction at F is removed, the pressure at F is momentarily greater than the external pressure by an amount ρgh. Liquid flows out of the tube with increasing velocity until p_F becomes equal to p_0. Suppose the liquid is then flowing through the tube with velocity v, and we assume (for simplicity of argument) that the cross sectional area of the vessel is very large compared with that of the tube, so that, by (2), we may neglect the downward velocity of the liquid in the vessel. If the pressure on the horizontal level of A of the (assumed static) liquid in the vessel is p_s, then the pressure at A just inside the entrance to the tube is $p_s - \tfrac{1}{2}\rho v^2$. The pressure difference across the entrance of the tube maintains the flow of liquid.

Obviously this explanation is greatly simplified, but it is in essence correct and can be refined to allow for the properties of real liquids and pipes. It is worth emphasizing that the explanation is unaltered whether p_0 is zero or finite.

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References

Abbott A F 1963 *Ordinary Level Physics* (London: Heinemann)

Ashhurst W 1966 *Introduction to Physics* (London: Murray)

Batchelor G K 1967 *An Introduction to Fluid Dynamics* (Cambridge: Cambridge University Press)

Briggs L J 1950 *J. appl. Phys.* **21** 721

Goldstein S 1938 *Modern Developments in Fluid Dynamics* Vol 1 (Oxford: Oxford University Press)

Massey B S 1968 *Mechanics of Fluids* (London: Van Nostrand)

Newman F H and Searle V H L 1957 *The General Properties of Matter* (London: Arnold)

Nokes M C 1948 *Sch. Sci. Rev.* **29** 233

Temperley H N V and Chambers Ll G 1946 *Proc. Phys. Soc.* **58** 420

Trevena D H 1967 *Contemp. Phys.* **8** 185